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A one dimensional free boundary problem describing swelling of a pocket of water in porous materials

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1 Introduction

This works is motivated by water swelling which occurs in each microscopic pore of concrete materials. Concrete materials has an infinite number of pores at microscopic level and the liquid of water exists in each pore. The pore is gradually filled by water due to the effects of the moisture content which distributes at macroscopic scale in the entire material. Such topics is strongly relevant in cold regions, where buildings exposed to extremely low temperatures undergo freezing and build microscopic ice lenses that ultimately lead to the mechanical damage of the material (cf. [20]). Our goal is to establish a two-scale model for moisture transport suitable for cementitious mixtures. In this paper, as the first investigation of our study, we propose a one-dimensional microscopic problem posed on a halfline with a moving boundary at one of the ends, and report the result which is concerned with the existence and uniqueness of a solution locally in time of this problem.

Here, we state the physical background of our free boundary problem describing water swelling. In this research we simplify one pore as a one dimensional halfline. Since we are interested in how far the water content can actually push a priori unknown moving boundary of swelling, we assume that pore depth is infinite although the actual physical length is finite. The timespan is $[0, T]$ and the pore is $[a, +\infty)$ with $a > 0$. Also, the boundary $z = a$ denotes the edge of the hole which touches wetness. The interval $[a, s(t)]$

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indicates the region of diffusion of the water content $u(t)$, where $s(t)$ is the moving interface of the water region, and $u(t)$ is distributed in the region denoted by $Q_s(T)$:

$$Q_s(T) := \{(t, z)|\ 0 < t < T,\ a < z < s(t)\}.$$

Based on the background, our free boundary problem, which denotes (P) := $(\mathrm{P})_{u_0,s_0,h}$, is formulated as follows by equations for the water content u and the front of the water content region s:

$$u_t - ku_{zz} = 0 \text{ for } (t, z) \in Q_s(T), \tag{1.1}$$
$$- u_z(t, a) = \beta(h - Hu(t, a))) \text{ for } t \in (0, T), \tag{1.2}$$
$$- u_z(t, s(t)) = u(t, s(t))s_t(t) \text{ for } t \in (0, T), \tag{1.3}$$
$$s_t(t) = a_0(u(t, s(t)) - \varphi(s(t))) \text{ for } t \in (0, T), \tag{1.4}$$
$$s(0) = s_0, u(0, z) = u_0(z) \text{ for } z \in [a, s_0]. \tag{1.5}$$

where k is a diffusion constant of the liquid of water, β is a given functions on $\mathbb{R}$ which is equal to 0 for negative input and takes a positive value for positive input, h is a given function on $[0, T]$, respectively, H and a_0 are given positive constants, φ is also given function on $\mathbb{R}$ and s_0 and u_0 are the initial data.

In our system, (1.1) is a diffusion equation of the water content u in the unknown region $[a, s]$, and the boundary condition (1.2) implies that the moisture content h inflows if h is present more than u at $z = a$. Also, the boundary condition (1.3) at $z = s(t)$ is obtained by the mass conservation law near the free boundary. Indeed, if the flux $u_z(t, a)$ at $z = a$ actives on the time interval $[t, t + \Delta t]$ for $t > 0$, namely, $s_t(t) > 0$, then, it holds that

$$\int_a^{s(t)} u(t, z)dz - ku_z(t, a)\Delta t = \int_a^{s(t+\Delta t)} u(t + \Delta t, z)dz.$$

Hence, by dividing Δt in both side and letting $\Delta t \to 0$ we obtain that

$$- ku_z(t, a) = \int_a^{s(t)} u_t(t, z)dz + s_t u(t, s(t)).$$

By $u_t = ku_{zz}$ in (1.1), we derive that

$$\begin{aligned} -ku_z(t, a) &= \int_a^{s(t)} ku_{zz}(t, z)dz + s_t u(t, s(t)) \\ &= ku_z(t, s(t)) - ku_z(t, a) + s_t u(t, s(t)). \end{aligned}$$

Thus, the boundary condition (1.3) at $z = s(t)$ can be obtained. Moreover, the ordinary differential equation (1.4) describes the growth rate of the free boundary s and it is determined by the balance between the water content $u(t, s(t))$ at $z = s(t)$ and $\varphi(s(t))$. The function $\varphi(s(t))$ represents the effect to suppress the growth of the free boundary, that is, we impose the breaking mechanism for the free boundary.

From the mathematical point of view, our free boundary problem resembles one phase Stefan problem for superheating, phase transitions, evaporation, and crystal dissolution and precipitation ([17, 18, 19, 21] and reference therein). Also, the existing works for the mathematical modeling of swelling are Fasano and collaborators [6, 7] and Zaal [22]. The main difference between these problems and our problem is the boundary conditions at the edge of the interval. In our problem we impose flux boundary conditions on both boundaries, while the homogeneous Dirichlet boundary condition is imposed on one of the boundary in the above problems. In particular, (1.2) is the condition described the strong connection between the moisture content h and the relative humidity u in one pore, and this is a significant feature of our free boundary problem to emphasize.

Also, from the viewpoint on free boundary problems in porous materials, Muntean and Böhm [14] originally proposed a free boundary problem as a mathematical model for concrete carbonation process in one dimension and Aiki and Muntean [3, 4, 5] proved the existence and uniqueness of a solution for a simplified Muntean-Böhm model and obtained the large time behavior of the free boundary at $t \to \infty$. Also, Sato et al [1, 16] proposed the following free boundary problem as a mathematical model of adsorption phenomenon in one pore:

$$\rho_g u_t - k u_{xx} = 0 \text{ for } (t, z) \in D_s(T), \tag{1.6}$$

$$u(t, L) = h(t) \text{ for } t \in (0, T), \tag{1.7}$$

$$k u_z(t, s(t)) = (\rho_a - \rho_g u(t, s(t))) s_t(t) \text{ for } t \in (0, T), \tag{1.8}$$

$$s_t(t) = \alpha(s(t), u(t, s(t))) \text{ for } t \in (0, T), \tag{1.9}$$

$$s(0) = s_0, u(0, z) = u_0(z) \text{ for } z \in [s_0, L], \tag{1.10}$$

where ρ_a is a constant of the density of the aqueous-H_2O, ρ_g is a constant of the amount of saturated water vapor, k is a diffusion constant of water in air, α is a Lipschitz continuous function on $\mathbb{R}^2$, h is a given boundary function on $[0, T]$, s_0 is a positive constant, and u_0 is a given function on $[s_0, L]$. In this study, they consider a one dimensional interval $[0, L]$ as one hole of the media, and $z = 0$ and $z = L$ represent the bottom of the hole and the top of the hole, respectively. Also, the interval $[0, s(t)]$ and $[s(t), L]$ indicate the water region and the air region, and u is the relative humidity in the hole distributed in $D_s(T) = \{(t, x) | \ 0 < t < T, s(t) < z < L\}$. This model is quite close to our model, and it is a significant feature of their model that the boundary condition (1.7) which represents that the relative humidity u has a direct contact with the moisture content h at the edge of the boundary. For the free boundary problem (1.6)-(1.10), Sato et al showed the existence of a solution locally in time. Also, Aiki and Murase [3] proved the existence of a solution globally in time of the above problem with $\alpha(s, u) = a(u - \varphi(s))$ in (1.8), where φ is a given function on $\mathbb{R}$ which represents the the rate from water-droplet to moisture based on an attractive force between the wall at $z = 0$ and the water-droplet, and also clarified the large time behavior of the solution as $t \to \infty$.

Recently, based on the results of Sato et al [16] and Aiki and Murase [2], Kumazaki et al [11] proposed a macro-micro model of moisture transport with adsorption phenomenon consisted of a diffusion equation for moisture in the entire material (macroscopic scale) and a free boundary problem describing adsorption phenomenon based on the system (1.6)-(1.9) in infinite pores (microscopic scale), and prove the local existence of a solution of this problem. In future, by using the idea of the two scale model we try to consider a two-scale model coupled with a diffusion equation of moisture distributed in the entire material and a free boundary problem describing water swelling in infinite microscopic pores. For this topics, we note in detail in Section 4.

Our paper is organized as follows: In Section 2, we state the assumptions and our main theorem concerned with the local existence and uniqueness of a solution for the free boundary problem. In Section 3, we introduce the outline of the proof of our main theorem without the detailed calculation. In Section 4, we note some remarks for the global existence of a solution of (P), and a two-scale problem for water swelling which consists a diffusion equation for the relative humidity distributed in the entire materials and a free boundary problem describing water swelling in infinite pores.

2 Assumptions

Throughout this paper, we assume the following restrictions on the model parameters and functions:

(A1) a, a_0, H, k and T are positive constants.

(A2) $h \in W^{1,2}(0,T) \cap L^\infty(0,T)$ with $h \geq 0$ on $(0,T)$.

(A3) $\beta \in C^1(\mathbb{R}) \cap W^{1,\infty}(\mathbb{R})$ such that $\beta = 0$ on $(-\infty, 0]$, and there exists $r_\beta > 0$ such that $\beta' > 0$ on $(0, r_\beta)$ and $\beta \equiv k_0$ on $[r_\beta, +\infty)$, where k_0 is a positive constant.

(A4) $\varphi \in C^1(\mathbb{R}) \cap W^{1,\infty}(\mathbb{R})$ such that $\varphi = 0$ on $(-\infty, 0]$, and there exists $r_\varphi > 0$ such that $\varphi' > 0$ on $(0, r_\varphi)$ and $\varphi \equiv c_0$ on $[r_\varphi, +\infty)$, where $0 < c_0 \leq \min\{2\varphi(a), |h|_{L^\infty(0,T)} H^{-1}\}$.

(A5) $s_0 > a$ and $u_0 \in H^1(a, s_0)$ such that $\varphi(s_0) \leq u_0(z) \leq |h|_{L^\infty(0,T)} H^{-1}$ on $[a, s_0]$.

For $T > 0$, let s be a function on $[0,T]$ and u be a function on $Q_s(T)$. Now, we define a solution to $(\mathrm{P})_{u_0,s_0,h}$ on $[0,T]$ in the following way.

Definition of solutions for (P): We call that pair (s,u) a solution to $(\mathrm{P})_{u_0,s_0,h}$ on $[0,T]$ if the following conditions (S1)-(S6) hold:

(S1) s, $s_t \in L^\infty(0,T)$, $a < s$ on $[0,T]$, $u \in L^\infty(Q_s(T))$, u_t, $u_{zz} \in L^2(Q_s(T))$ and $t \in [0,T] \to |u_z(t,\cdot)|_{L^2(a,s(t))}$ is bounded;

(S2) $u_t - ku_{zz} = 0$ on $Q_s(T)$;

(S3) $-ku_z(t,a) = \beta(h(t) - Hu(t,a))$ for a.e. $t \in [0,T]$;

(S4) $-ku_z(t,s(t)) = u(t,s(t))s_t(t)$ for a.e. $t \in [0,T]$;

(S5) $s_t(t) = a_0(u(t,s(t)) - \varphi(s(t)))$ for a.e. $t \in [0,T]$;

(S6) $s(0) = s_0$ and $u(0,z) = u_0(z)$ for $z \in [a, s_0]$.

Our main result of this paper is the existence and uniqueness of a time local solution for the problem $(\mathrm{P})_{u_0,s_0,h}$.

Theorem 1 (cf. [12]) *Let $T > 0$. If (A1)-(A5) hold, then there exists $T^* > 0$ such that $(P)_{u_0,s_0,h}$ has a unique solution (s,u) on $[0,T^*]$ satisfying $\varphi(a) \le u \le |h|_{L^\infty(0,T)}H^{-1}$ a.e. on $Q_s(T^*)$.*

To prove Theorem 1, we transform $(\mathrm{P})_{u_0,s_0,h}$, initially posed in a non-cylindrical domain, to a cylindrical domain. Let $T > 0$. For given $s \in W^{1,2}(0,T)$ with $a < s(t)$ on $[0,T]$, we introduce the following new function obtained by the indicated change of variables:

$$\tilde{u}(t,y) = u(t,(1-y)a + ys(t)) \text{ for } (t,y) \in Q(T) := (0,T) \times (0,1).$$

By using the function $\tilde{u}$, $(\mathrm{P})_{\tilde{u}_0,s_0,h}$ becomes:

$$\tilde{u}_t - \frac{k}{(s(t)-a)^2}\tilde{u}_{yy} = \frac{ys_t}{s(t)-a}\tilde{u}_y \text{ for } (t,z) \in Q(T), \tag{2.1}$$

$$-\frac{k}{s(t)-a}\tilde{u}_y(t,0) = \beta(h - H\tilde{u}(t,0)) \text{ for } t \in (0,T), \tag{2.2}$$

$$-\frac{k}{s(t)-a}\tilde{u}_y(t,1) = u(t,s(t))s_t(t) \text{ for } t \in (0,T), \tag{2.3}$$

$$s_t = a_0(\tilde{u}(t,1) - \varphi(s(t)) \text{ for } t \in (0,T), \tag{2.4}$$

$$s(0) = s_0, \tag{2.5}$$

$$\tilde{u}(0,y) = \tilde{u}(0,y) = u_0(1-y)a + ys(0))(:= \tilde{u}_0(y)) \text{ for } y \in [0,1]. \tag{2.6}$$

Definition of solutions for $(\mathbf{P})_{\tilde{u}_0,s_0,h}$: For $T > 0$, let s be functions on $[0,T]$ and $\tilde{u}$ be a function on $Q(T)$, respectively. We call that a pair $(s,\tilde{u})$ is a solution of $(\mathrm{P})_{\tilde{u}_0,s_0,h}$ on $[0,T]$ if the conditions (S'1)-(S'2) hold:

(S'1) s, $s_t \in L^\infty(0,T)$, $a < s$ on $[0,T]$, $\tilde{u} \in W^{1,2}(Q(T)) \cap L^\infty(0,T;H^1(0,1)) \cap L^2(0,T;H^2(0,1)) \cap L^\infty(Q(T))$.

(S'2) (2.1)–(2.6) hold.

For $(\mathrm{P})_{\tilde{u}_0,s_0,h}$, we note that the following theorem which is concerned with the existence and uniqueness of solutions holds:

Theorem 2 (cf. [12]) *Let $T > 0$. If (A1)-(A5) hold, then there exists $T^* > 0$ such that $(P)_{\tilde{u}_0,s_0,h}$ has a unique solution $(s,\tilde{u})$ on $[0,T^*]$.*

By Theorem 2, we see that for a solution $(s,\tilde{u})$ of $(\mathrm{P})_{\tilde{u}_0,s_0,h}$ on $[0,T^*]$, a pair of the function (s,u) with the variable

$$u(t,z) := \tilde{u}\left(t, \frac{z-a}{s(t)-a}\right) \text{ for } z \in [a,s(t)] \tag{2.7}$$

is a solution of $(\mathrm{P})_{\tilde{u}_0,s_0,h}$. Finally, by proving that (s,u) satisfies $\varphi(a) \le u \le |h|_{L^\infty(0,T)}H^{-1}$ on $Q_s(T^*)$, the pair (s,u) is the desired solution satisfying Theorem 1.

3 Outline of the proof

In this section, we restrict ourselves only the outline of the proof, so we omit the detailed calculation. First, we consider the following problem $(\mathrm{AP})^{\sigma}_{\tilde{u}_0,s,h}$ for given s:

$$\tilde{u}_t(t,y) - \frac{k}{(s(t)-a)^2}\tilde{u}_{yy}(t,y) = \frac{ys_t(t)}{s(t)-a}\tilde{u}_y(t,y) \text{ for } (t,y) \in Q(T), \tag{3.1}$$

$$-\frac{k}{s(t)-a}\tilde{u}_y(t,0) = \beta(h(t) - H\tilde{u}(t,0)) \text{ for } t \in (0,T), \tag{3.2}$$

$$-\frac{k}{s(t)-a}\tilde{u}_y(t,1) = a_0\sigma(\tilde{u}(t,1))(\sigma(\tilde{u}(t,1)) - \varphi(s(t))) \text{ for } t \in (0,T), \tag{3.3}$$

$$\tilde{u}(0,y) = \tilde{u}_0(y) \text{ for } y \in [0,1], \tag{3.4}$$

where and σ is a lower cut-off function on $\mathbb{R}$ given by

$$\sigma(r) = \begin{cases} r & \text{if } r > \varphi(a), \\ \varphi(a) & \text{if } r \le \varphi(a). \end{cases}$$

.

In the proof of the existence of solutions, we use the abstract theory of evolution equations in Hilbert spaces governed by time-dependent subdifferentials which is characterized by the following form (cf. [10] and references cited therein):

$$u_t(t) + \partial\varphi^t(u(t)) \ni l(t) \text{ in } H \text{ for } t \in [0,T] \tag{3.5}$$

where φ^t is a proper, lower semi-continuous, convex function on Hilbert spaces H for $t \in [0,T]$, and $\partial\varphi^t$ is the subdifferential of φ^t defined by

$$\partial\varphi^t(u) := \{z^* \in H \mid (z^*, v-u)_H \le \varphi^t(v) - \varphi^t(u) \text{ for } v \in H\},$$

and l is a given H-valued function on $[0,T]$. For $(\mathrm{AP})_{\tilde{u}_0,s,h}$, by setting φ^t on $H = L^2(0,1)$ defined by

$$\varphi^t(u) := \begin{cases} \dfrac{k}{2(s(t)-a)^2}\displaystyle\int_0^1 |u_y(y)|^2 dy + \dfrac{1}{s(t)-a}\displaystyle\int_0^{u(1)} a_0\sigma(\xi)(\sigma(\xi) - \varphi(s(t)))d\xi \\ -\dfrac{1}{s(t)-a}\displaystyle\int_0^{u(0)} \beta(h(t) - H\xi)d\xi \text{ if } u \in D(\psi^t), \\ +\infty \text{ if } \mathit{otherwise}, \end{cases}$$

where $D(\varphi^t) = \{z \in H^1(0,1) | z \ge 0 \text{ on } [0,1]\}$ for $t \in [0,T]$, and considering $\frac{ys_t(t)}{s(t)-a}\tilde{u}_y(t)$ with given $s \in W^{1,\infty}(0,T)$ as $l(t)$, we apply (3.5). As a property of the function φ we note that its subdifferential realizes the second term in the left hand side of (3.1) with the boundary conditions (3.2) and (3.3).

For $s \in W^{1,2}(0,T)$, we take a sequence $\{s_n\} \subset W^{1,\infty}(0,T)$ such that $s_n \to s$ in $W^{1,2}(0,T)$ as $n \to \infty$, and we consider the limiting process with respect to n after obtaining some energy estimates of $\tilde{u}_n$ independent of n, where $\tilde{u}_n$ is a solution on $[0,T]$ of $(\mathrm{AP})^{\sigma}_{\tilde{u}_0,s_n,h}$ for each n. Then, we can find a solution $\tilde{u}$ of $(\mathrm{AP})^{\sigma}_{\tilde{u}_0,s,h}$ on $[0,T]$ for given $s \in W^{1,2}(0,T)$.

Next, we define the set

$$M(T,s_0,a') := \{s \in W^{1,2}(0,T) | a' \le s < L \text{ on } [0,T], s(0) = s_0\}.$$

Also, for given $s \in M(T,s_0,a')$, we define the operator $\Phi : M(T,s_0,a') \to V(T) := W^{1,2}(0,T;L^2(0,1)) \cap L^\infty(0,T;H^1(0,1))$ by $\Phi(s) = \tilde{u}$, where $\tilde{u}$ is a solution of $(\mathrm{AP})^{\sigma}_{\tilde{u}_0,s,h}$, and the operator $\Gamma_T : M(T,s_0,a') \to W^{1,2}(0,T)$ by $\Gamma_T(s) = s_0 + \int_0^t a_0(\sigma(\Phi(s)(\tau,1)) - \varphi(s(\tau)))d\tau$ for $t \in [0,T]$. Moreover, for any $K > 0$ we put

$$M_K(T) := \{s \in M(T,s_0,a') | \ |s|_{W^{1,2}(0,T)} \le K\}.$$

The solution of $(\mathrm{P})_{\tilde{u}_0,s_0,h}$ is obtained by the following procedure: First, by the continuous dependence of a solution $\tilde{u}$ of $(\mathrm{AP})^{\sigma}_{\tilde{u}_0,s,h}$ for given s in a suitable subspace of $W^{1,2}(0,T)$ we show that Γ_{T_1} is a contraction mapping on $M_K(T_1)$ in $W^{1,2}(0,T_1)$ for some $T_1 < T$. Next, by Banach's fixed point theorem, we prove the existence of a time local solution of $(\mathrm{P})^{\sigma}_{\tilde{u}_0,s_0,h}$, where $(\mathrm{P})^{\sigma}_{\tilde{u}_0,s_0,h}$ is a problem replaced $\tilde{u}(t,1)$ by $\sigma(\tilde{u}(t,s(t))$. Therefore, we can prove that Theorem 2 holds.

Finally, by using (2.7), the solution of $(\mathrm{P})^{\sigma}_{\tilde{u}_0,s_0,h}$ is a solution of $(\mathrm{P})^{\sigma}_{u_0,s_0,h}$, and by the maximum principle, we observe that a solution (s,u) of $(\mathrm{P})^{\sigma}_{u_0,s_0,h}$ on $[0,T]$ satisfies $\varphi(a) \le u \le |h|_{L^\infty(0,T)} H^{-1}$ on $Q(T)$, and remove σ.

4 Further topics

4.1 Global solutions

To obtain a globally in time solution of $(\mathrm{P})_{u_0,s_0,h}$, we challenge to extend the existing locally in time solution to $(\mathrm{P})^{\sigma}_{\tilde{u}_0,s_0,h}$. However, if the free boundary s equals to the edge of the hole a, then $s - a = 0$, that is, there is no domain to find a solution. Therefore, we need to guarantee that s is strictly grater than a at the maximal existence time. Since the free boundary s is not always monotone with respect to time t, it is not easy to prove such a strict lower bound on the sharp interface position. In the forthcoming paper [13], by assuming that the initial data u_0 is small in some sense and is strictly grater than $\varphi(a)$, we guarantee that the free boundary s is strictly grater than $\varphi(a)$, and show the existence and uniqueness of a globally in time solution of $(\mathrm{P})_{u_0,s_0,h}$.

4.2 Two-scale model

As the mentioned in Introduction, our long-term goal is to construct a two-scale model for moisture transport suitable for cementitious mixtures. The main idea of so-called two scale model is established in [9], and is applied in [8, 11, 15]. Based on this idea, we challenge to consider a two-scale model describing water swelling in porous materials, where at the macroscopic scale the transport of moisture follows a porous-media-like equation, while at the microscopic scale the moisture is involved in an adsorption-desorption process leading to a strong local swelling of the pores. One of the expected problem is:

$$h_t - \Delta h = \beta(h(t,x) - Hu(t,x,a(x)) \text{ for } (t,x) \in (0,T) \times \Omega, \tag{4.1}$$

$$u_t - ku_{zz} = 0 \text{ on } \tilde{Q}_s(T) \tag{4.2}$$

$$-u_z(t,x,a(x)) = \beta(h(t,x) - Hu(t,x,a(x))) \text{ for } (t,x) \in (0,T) \times \Omega, \tag{4.3}$$

$$-u_z(t,x,s(t,x)) = u(t,x,s(t,x))s_t(t,x) \text{ for } (t,x) \in (0,T) \times \Omega, \tag{4.4}$$

$$s_t(t,x) = a_0(u(t,x,s(t,x)) - \varphi(s(t,x))) \text{ for } (t,x) \in (0,T) \times \Omega. \tag{4.5}$$

$$h(0,x) = h_0(x), s(0,x) = s_0(x) \text{ for } x \in \Omega, \tag{4.6}$$

$$u(0,x,z) = u_0(x,z) \text{ for } (x,z) \in \Omega \times [a, s_0], \tag{4.7}$$

where Ω is a bounded smooth domain in $\mathbb{R}^3$ which is occupied by cementitious materials, and h represents the relative humidity distributed in Ω, and β, k, H and a_0 are the same function and positive constants as in (P). Also, h_0, s_0 are given function on Ω and u_0 is also given function on $\Omega \times [a, s_0(x)]$.

As the above model, we consider a macro domain Ω and a micro domain for each $x \in \Omega$. In the macro domain Ω, we consider the diffusion equation (4.1) of moisture. On the other hand, we regard that the micro domain is one pore of the material at each $x \in \Omega$, and each pore is a one dimensional halfline $[a(x), +\infty]$. Also, we suppose that the halfline has the water content region $[a(x), s(t,x)]$ and a priori unknown swelling space $[s(t,x), +\infty]$, where $s = s(t,x)$ is the front of the water content region in the halfline at each $x \in \Omega$, and the water content $u = u(t,x,z)$ is distributed in $\tilde{Q}_s(T) := \{(t,x,z) | (t,x) \in (0,T) \times \Omega, a(x) < z < s(t,x)\}$. The system (4.2)-(4.5) corresponds to the free boundary problem (1.1)-(1.4) for each $x \in \Omega$, and (4.2)-(4.5) indicates that (1.1)-(1.4) holds in each pore for each $x \in \Omega$. In these settings, we consider infinite numbers of the free boundary problems all at once. As one of the significant feature of our model, by $\beta(h(t,x) - Hu(t,x,s(t,x))$ in (4.1) and (4.3) we consider the connection between the macro domain and the micro domain, and attempt to consider how the structure on one of the scale affects the structure on another scale. The constant H is called upscalling constant, and by this constant H we can consider the microscopic function at macroscopic scale. Thus, our two scale model consists of a diffusion equation of the relative humidity at macroscopic scale and an infinite free boundary problems describing water swelling at microscopic scale. In the future, we clarify the dependence between the relative humidity h at macroscopic scale and the water content u at microscopic scale, and build and study a model for the transport of moisture suitable for cementitious mixtures.

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